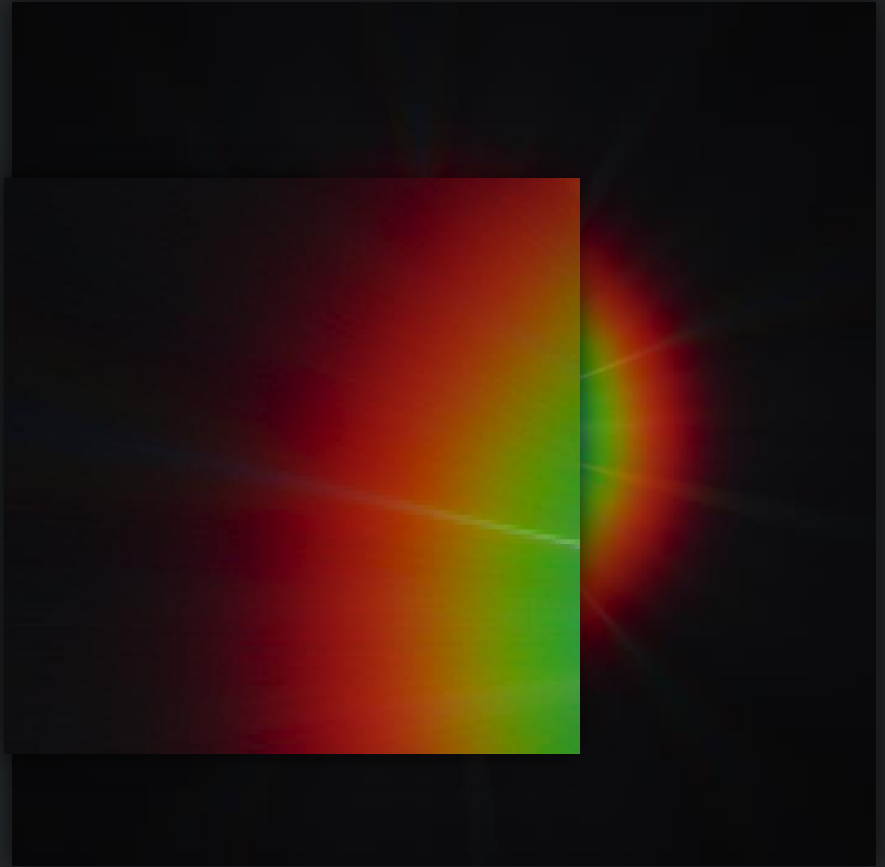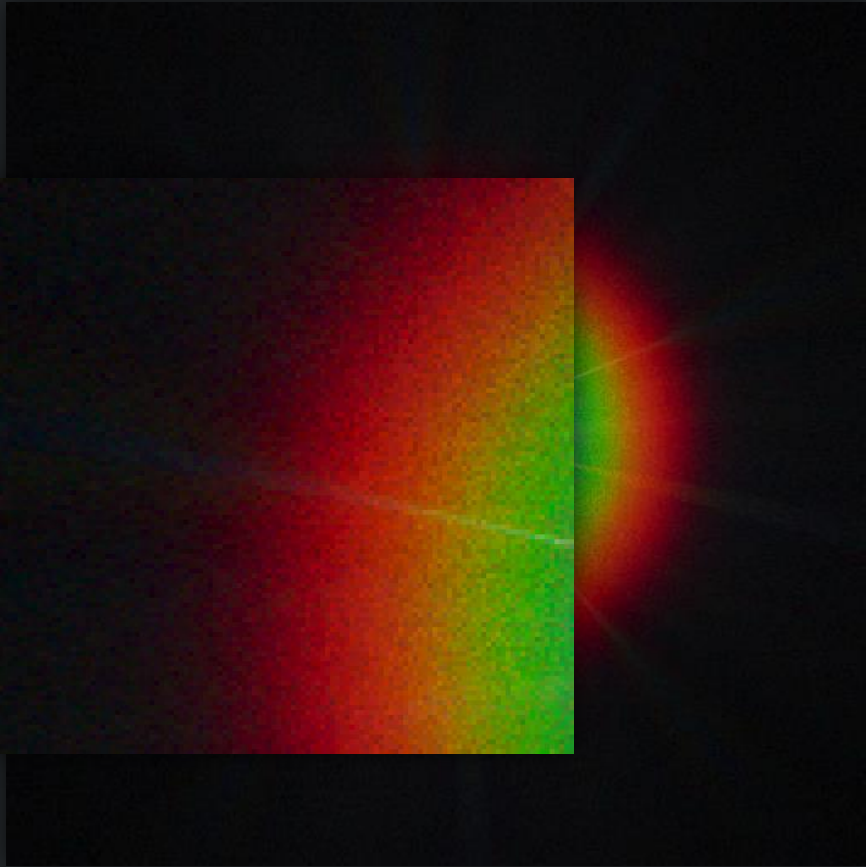# SPECTRAL RAY DIFFERENTIALS

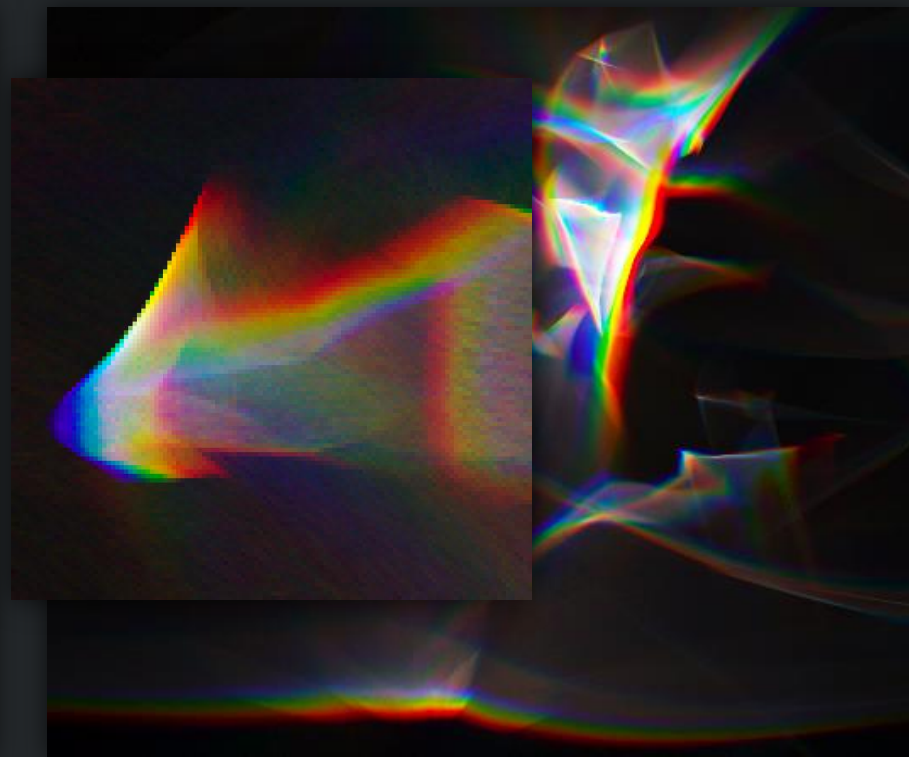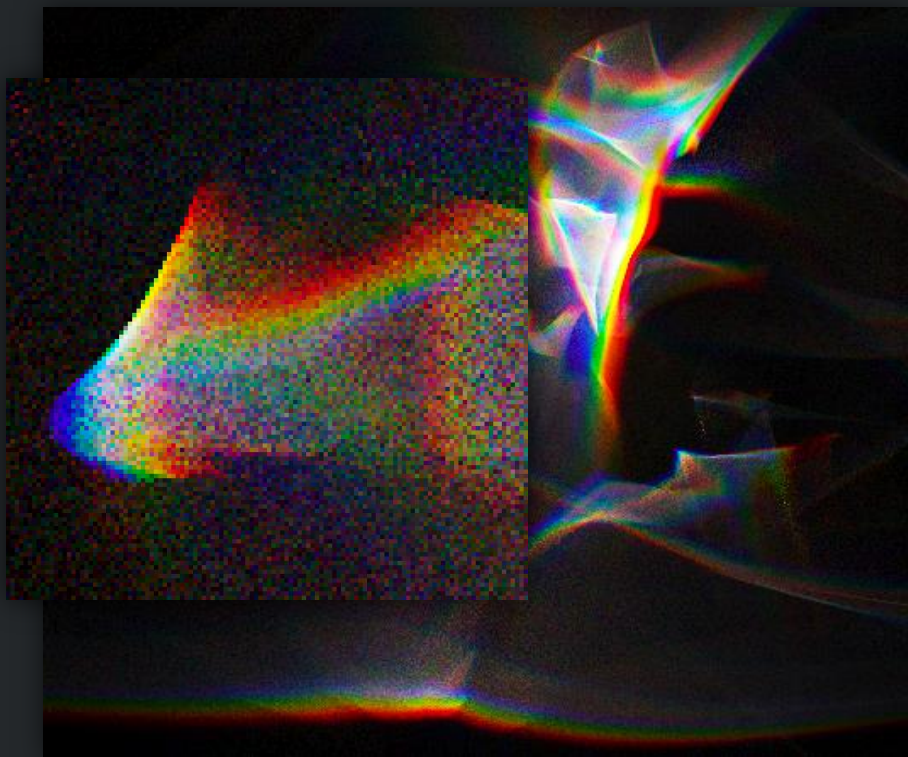**Oskar Elek** [1,2,3]   Tobias Ritschel [1,2,3]   Pablo Bauszat [4]

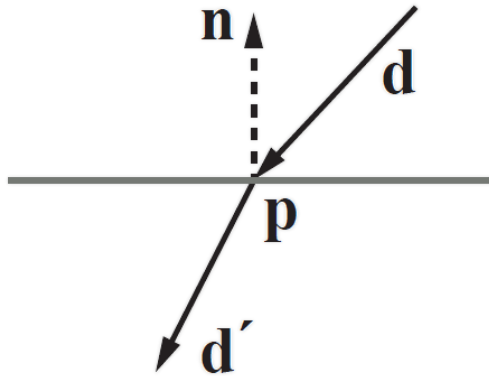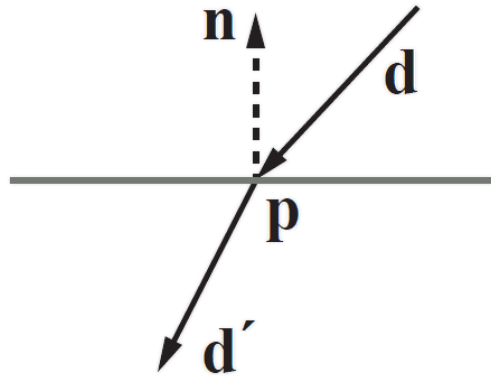Marcus Magnor [4]   Hans-Peter Seidel [1,2,3]

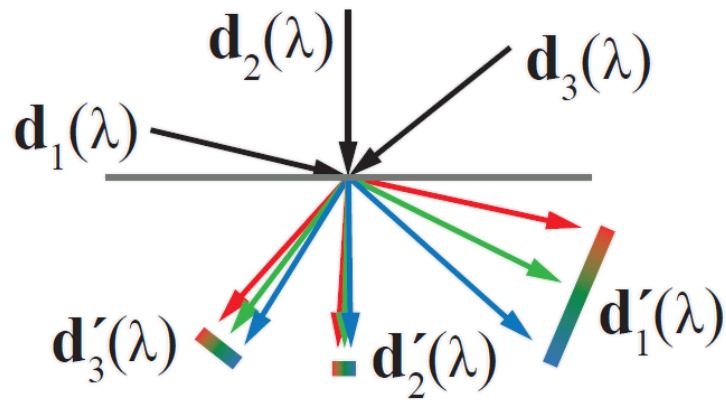# TEASER

TEASER

DISPERSIVE REFRACTION

Snell's law:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$
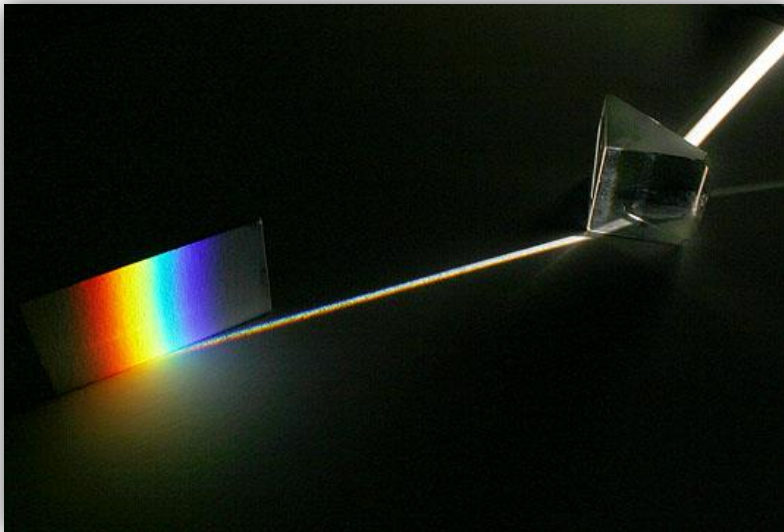
DISPERSIVE REFRACTION

Snell's law:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

DISPERSIVE REFRACTION

Credit: Andrew Davidhazy

Credit: Peter Kutz

DISPERSIVE REFRACTION
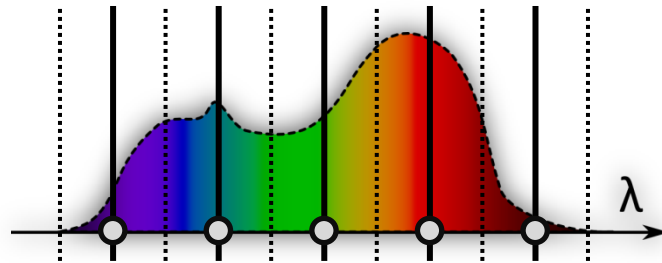

Credit: Bad Mobile Phone Camera ™

DISPERSION RENDERING 101
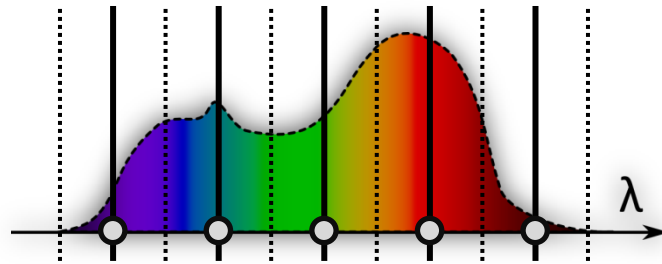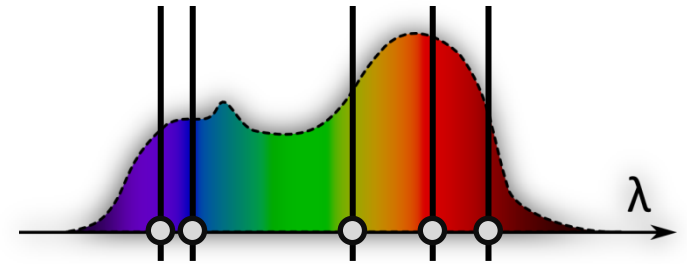
Regular sampling
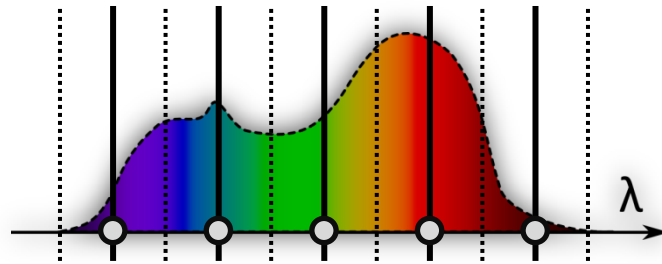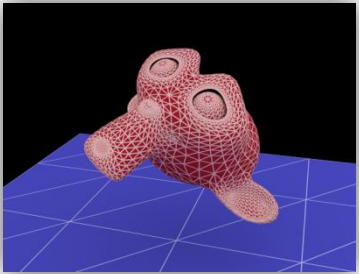
DISPERSION RENDERING 101

Regular sampling

Stochastic sampling
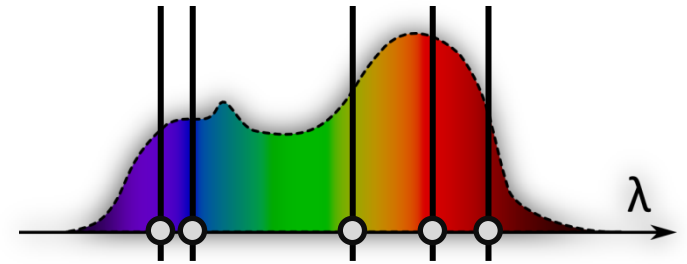
DISPERSION RENDERING 101

Regular sampling

Stochastic sampling

DISPERSION RENDERING 101
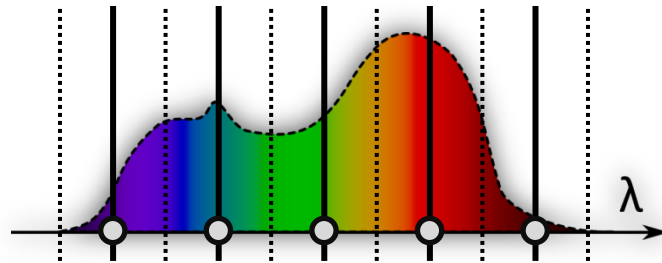
Regular sampling   Stochastic sampling
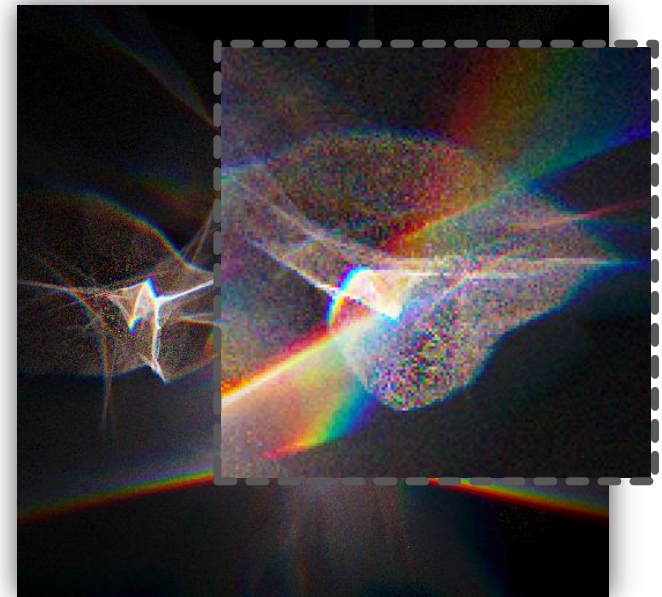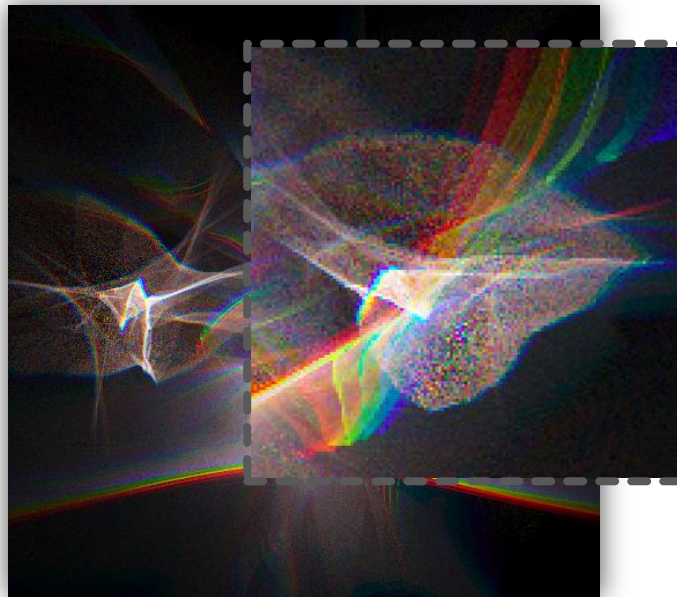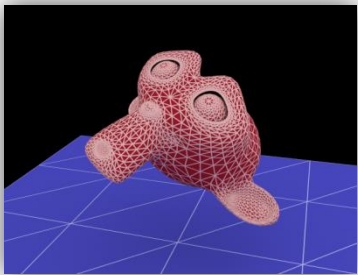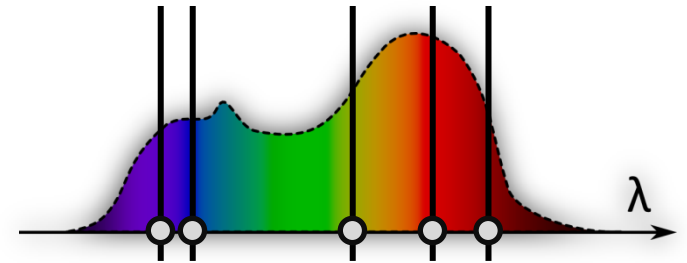
DISPERSION RENDERING 101

- Additional concerns
  - Requires spectral rendering
  - Tracing monochromatic rays

DISPERSION RENDERING 101

- Additional concerns
  - Requires spectral rendering
  - Tracing monochromatic rays

- Reconstruction

DISPERSION RENDERING 101

- Additional concerns
  - Requires spectral rendering
  - Tracing monochromatic rays
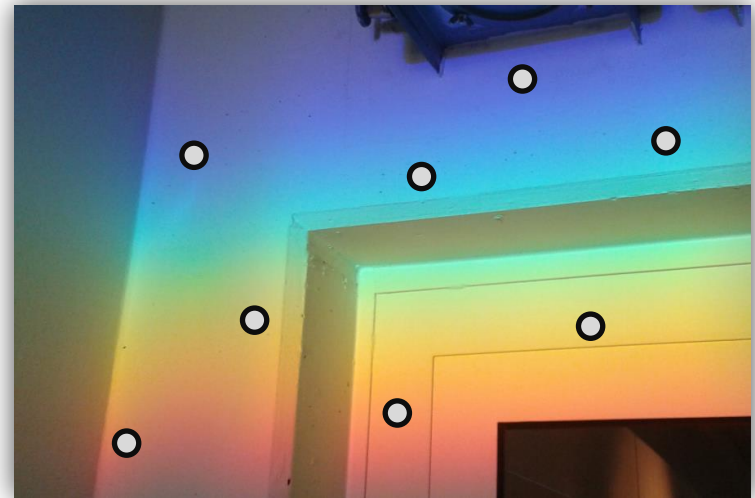
- Reconstruction

DISPERSION RENDERING 101

- Additional concerns
  - Requires spectral rendering
  - Tracing monochromatic rays

- **Reconstruction**

RAY DIFFERENTIALS

- Introduced by Igehy in 1999

- Later extended to:
  - Paths (Suykens, Willems; 2001)
  - Photons (Schjoth, Frisvad, Erleben, Sporring; 2007)
  - Diffuse reflection (Fabianowski, Dingliana; 2009)

Ray Differentials

- Introduced by Igehy in 1999

- Later extended to:

  – Paths (Suykens, Willems; 2001)

  – Photons (Schjoth, Frisvad, Erleben, Sporring; 2007)

  – Diffuse reflection (Fabianowski, Dingliana; 2009)

$$\frac{\partial \mathbf{R}}{\partial s} = \left(\frac{\partial \mathbf{p}}{\partial s}, \frac{\partial \mathbf{d}}{\partial s} \right)$$
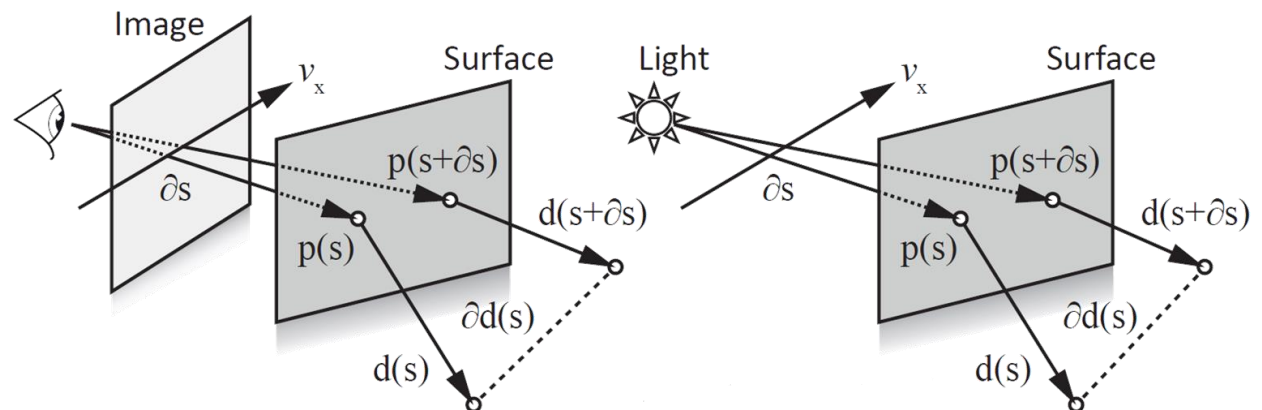
RAY DIFFERENTIALS

- Introduced by Igehy in 1999
- Later extended to:
  - Paths (Suykens, Willems; 2001)
  - Photons (Schjoth, Frisvad, Erleben, Sporring; 2007)
  - Diffuse reflection (Fabianowski, Dingliana; 2009)

$$\frac{\partial \mathbf{R}}{\partial s} = \left(\frac{\partial \mathbf{p}}{\partial s}, \frac{\partial \mathbf{d}}{\partial s} \right)$$
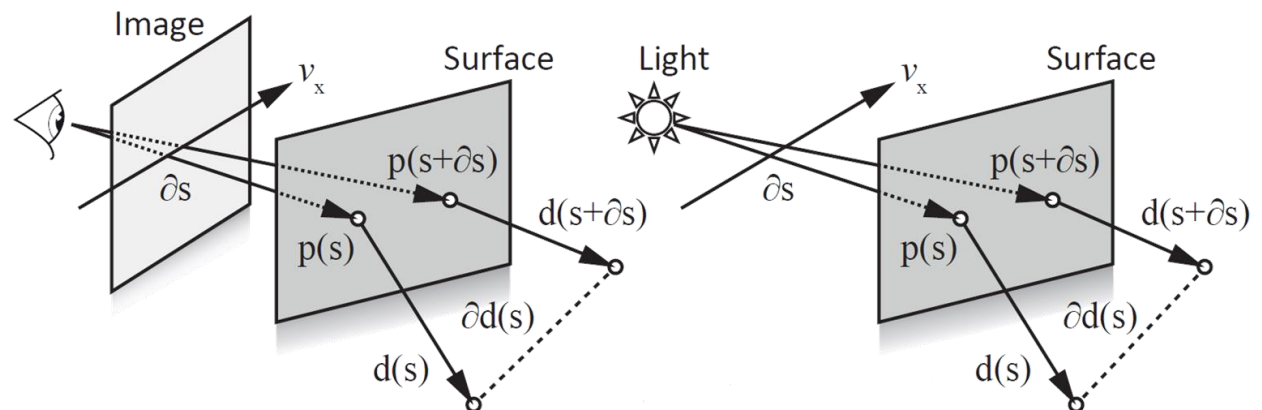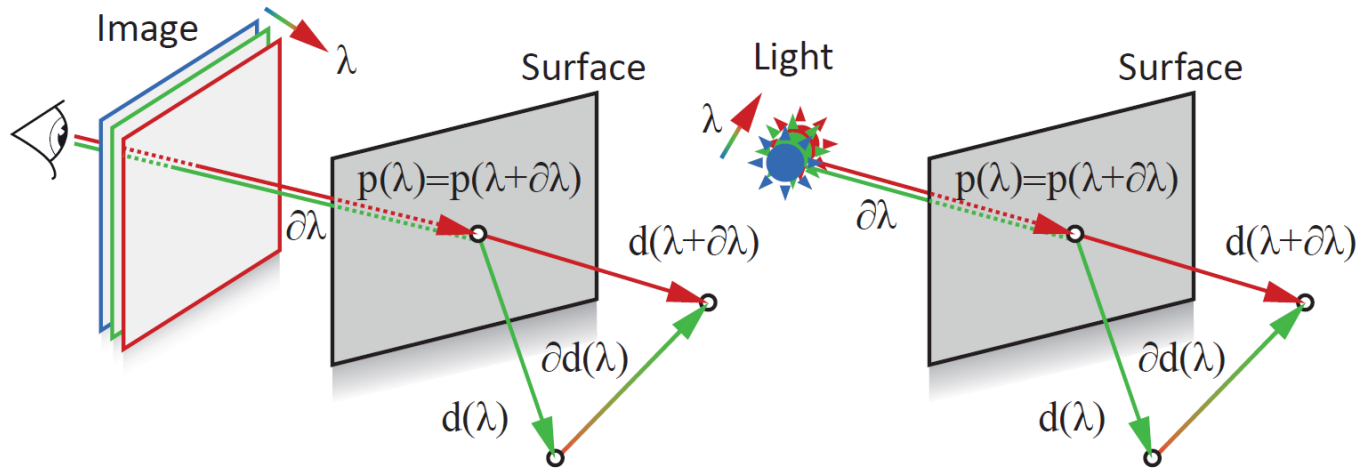
RAY DIFFERENTIALS

- Introduced by Igehy in 1999

- Later extended to:

  - Paths (Suykens, Willems; 2001)

  - Photons (Schjoth, Frisvad, Erleben, Sporring; 2007)

  - Diffuse reflection (Fabianowski, Dingliana; 2009)

$$\frac{\partial \mathbf{R}}{\partial s} = \left(\frac{\partial \mathbf{p}}{\partial s}, \frac{\partial \mathbf{d}}{\partial s} \right)$$

$$\frac{\partial \mathbf{R}}{\partial t} = \left(\frac{\partial \mathbf{p}}{\partial t}, \frac{\partial \mathbf{d}}{\partial t} \right)$$
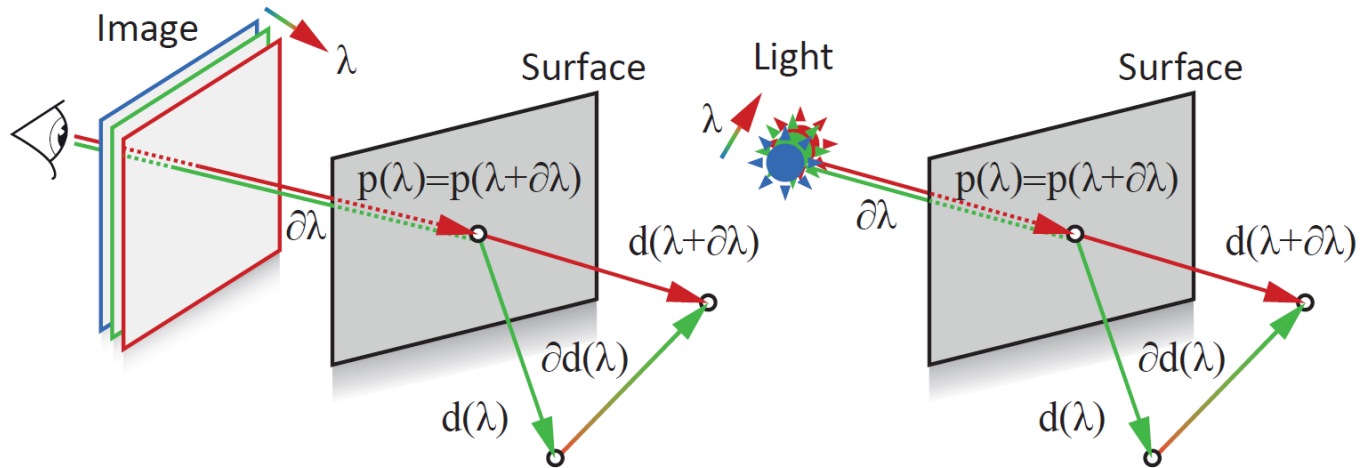
SPECTRAL RAY DIFFERENTIALS

$$\frac{\partial \mathbf{R}}{\partial \lambda} = \left(\frac{\partial \mathbf{p}}{\partial \lambda}, \frac{\partial \mathbf{d}}{\partial \lambda} \right)$$
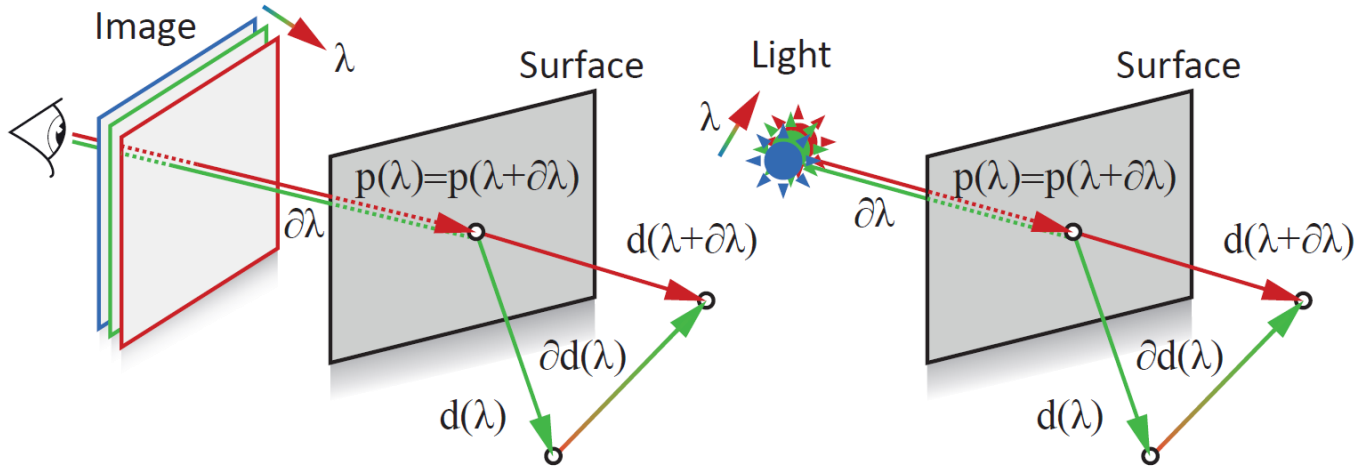
SPECTRAL RAY DIFFERENTIALS

$$\frac{\partial \mathbf{R}}{\partial \lambda} = \left(\frac{\partial \mathbf{p}}{\partial \lambda}, \frac{\partial \mathbf{d}}{\partial \lambda} \right)$$
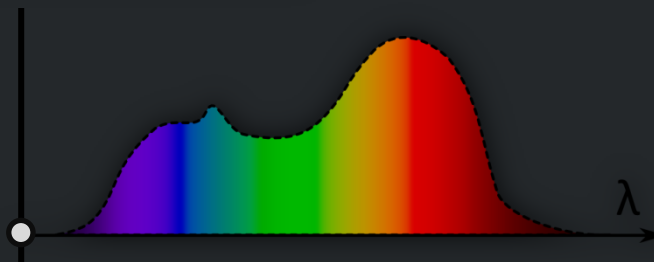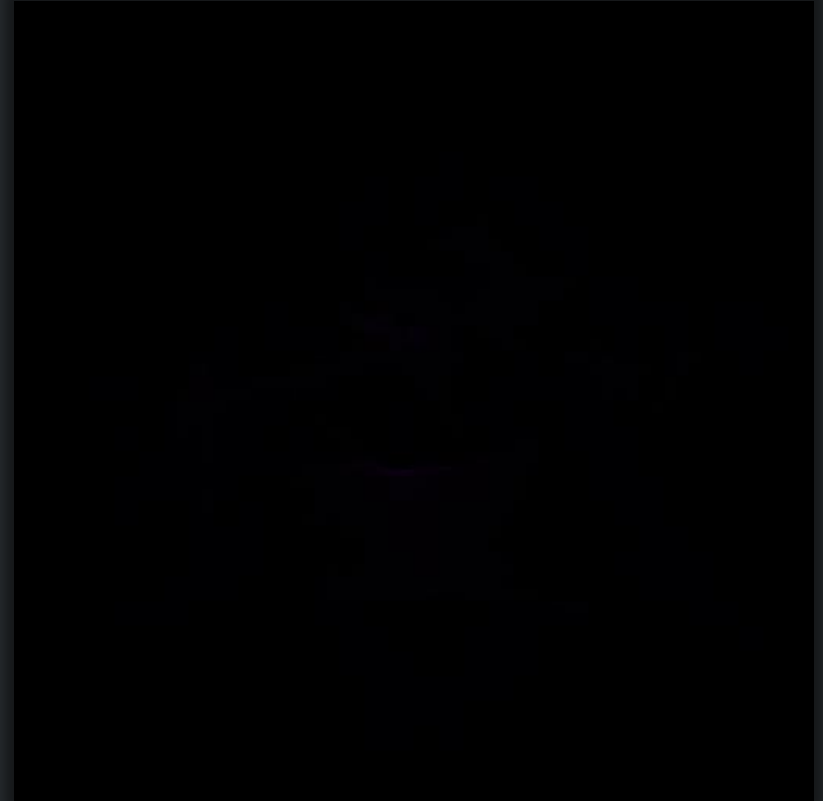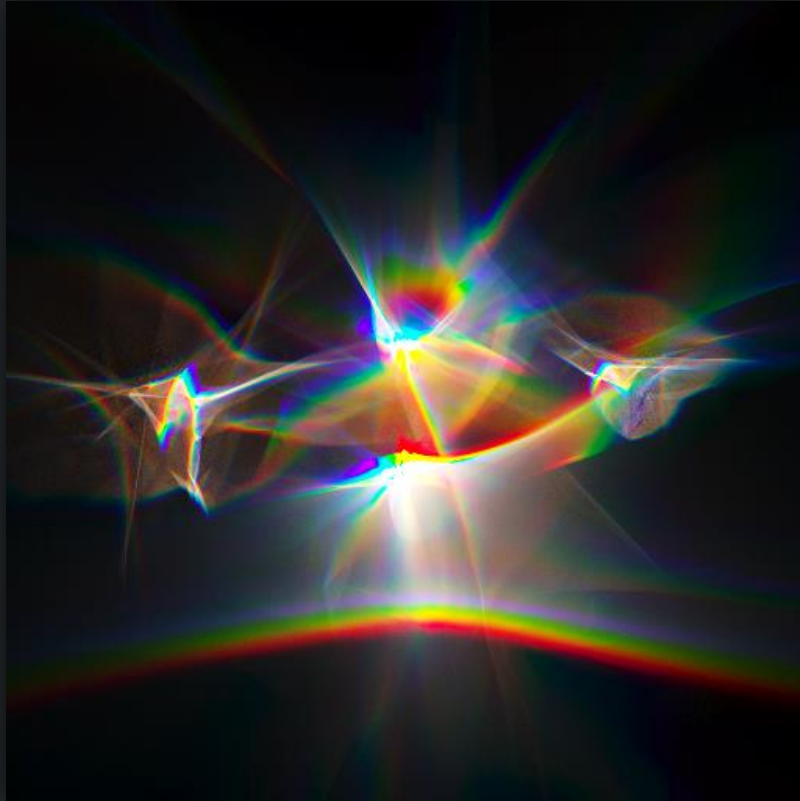
$$\frac{\partial \mathbf{d}'}{\partial \lambda} = \frac{\partial \eta}{\partial \lambda}\mathbf{d} + \eta\frac{\partial \mathbf{d}}{\partial \lambda} - \frac{\partial \mu}{\partial \lambda}\mathbf{n} - \mu\frac{\partial \mathbf{n}}{\partial \lambda}$$

```
vec3 ∂n = normalDifferential(∂p);
float θ = dot(d,n);
float ω = sqrt(1 - sqr(η) + sqr(η)*sqr(θ));
float μ = η*θ + ω;
float ∂η = etaDifferential(η);
float ∂t = dot(∂d,n) + dot(d,∂n);
float ∂O = (-η*∂η + η*∂η*sqr(θ) + sqr(η)*θ*∂t)/ω;
float ∂μ = ∂η*θ + η*∂t + ∂O;
return ∂η*d + η*∂d - ∂μ*n - μ*∂n;
```

SPECTRAL RAY DIFFERENTIALS
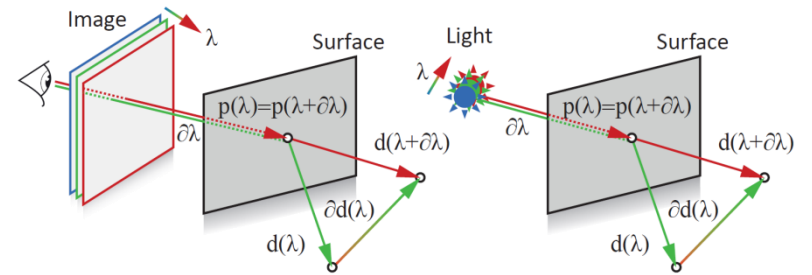

$$\frac{\partial \mathbf{R}}{\partial \lambda} = \left(\frac{\partial \mathbf{p}}{\partial \lambda}, \frac{\partial \mathbf{d}}{\partial \lambda} \right)$$

$$\frac{\partial \mathbf{d}'}{\partial \lambda} = \frac{\partial \eta}{\partial \lambda}\mathbf{d} + \eta\frac{\partial \mathbf{d}}{\partial \lambda} - \frac{\partial \mu}{\partial \lambda}\mathbf{n} - \mu\frac{\partial \mathbf{n}}{\partial \lambda}$$

```
vec3 ∂n = normalDifferential(∂p);
float θ = dot(d,n);
float ω = sqrt(1 - sqr(η) + sqr(η)*sqr(θ));
float μ = η*θ + ω;
float ∂η = etaDifferential(η);
float ∂t = dot(∂d,n) + dot(d,∂n);
float ∂O = (-η*∂η + η*∂η*sqr(θ) + sqr(η)*θ*∂t)/ω;
float ∂μ = ∂η*θ + η*∂t + ∂O;
return ∂η*d + η*∂d - ∂μ*n - μ*∂n;
```

SPECTRAL RAY DIFFERENTIALS

APPLICATION

- Tracing
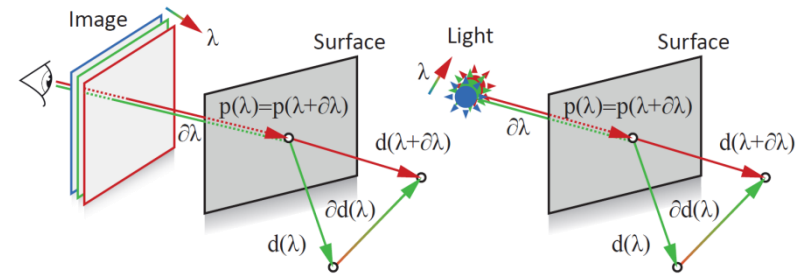 - Identical to ray differentials

APPLICATION

- Tracing
  - Identical to ray differentials
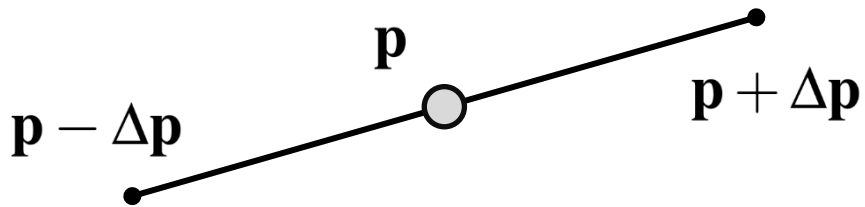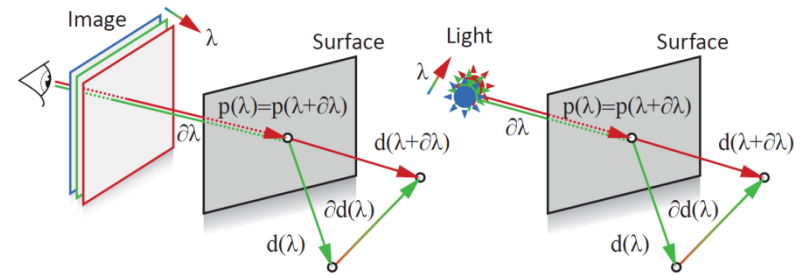- Reconstruction
  - 1st order approximation

$$\mathbf{R}(\lambda + \Delta\lambda) - \mathbf{R}(\lambda) \approx \Delta\lambda \frac{\partial \mathbf{R}(\lambda)}{\partial \lambda}$$
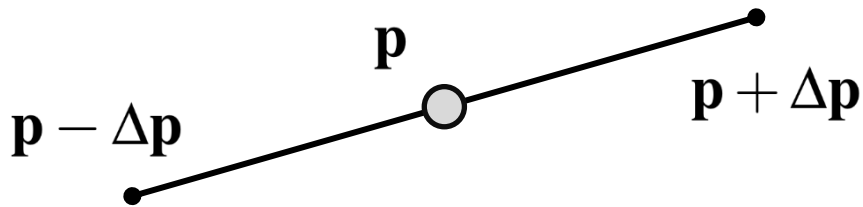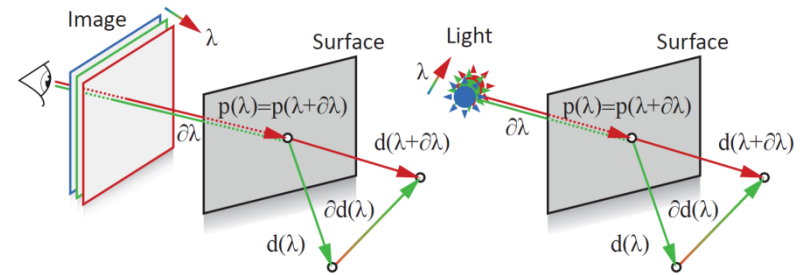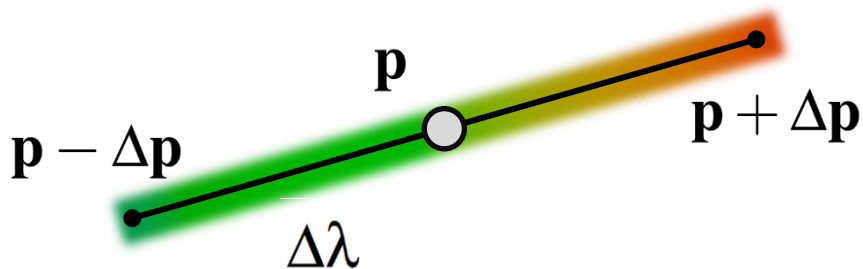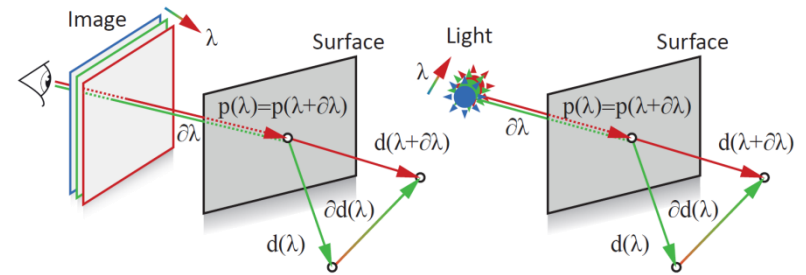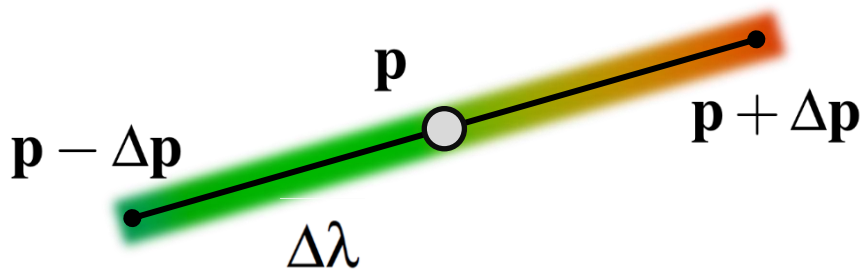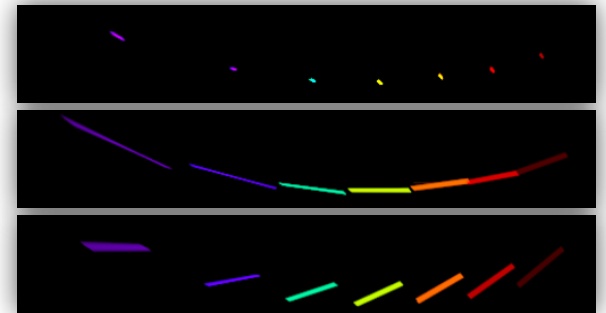
APPLICATION

- Tracing
  - Identical to ray differentials
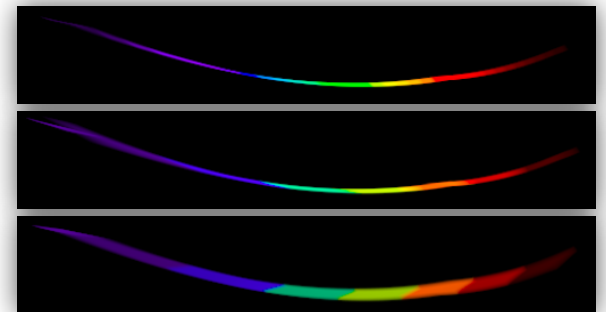- Reconstruction
  - 1st order approximation

$$\mathbf{R}(\lambda + \Delta\lambda) - \mathbf{R}(\lambda) \approx \Delta\lambda \frac{\partial \mathbf{R}(\lambda)}{\partial \lambda}$$

$\mathbf{p}$

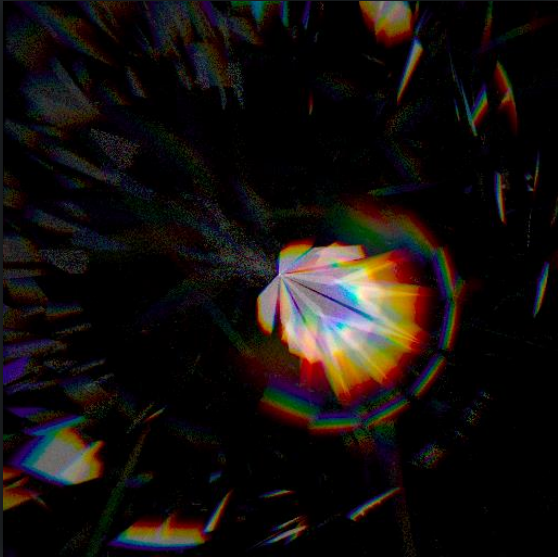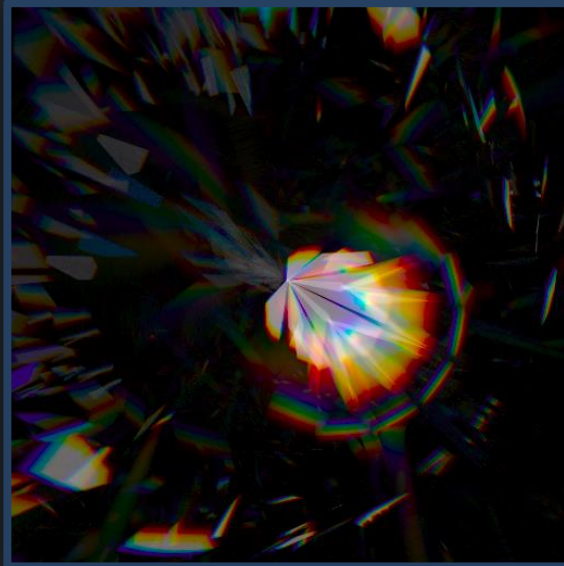$\mathbf{p} - \Delta\mathbf{p}$ $\mathbf{p} + \Delta\mathbf{p}$

APPLICATION

- Tracing
  - Identical to ray differentials
- Reconstruction
  - 1st order approximation

$$\mathbf{R}(\lambda + \Delta\lambda) - \mathbf{R}(\lambda) \approx \Delta\lambda \frac{\partial \mathbf{R}(\lambda)}{\partial \lambda}$$

$\mathbf{p}$

$\mathbf{p} - \Delta\mathbf{p}$

$\mathbf{p} + \Delta\mathbf{p}$

APPLICATION

- Tracing
  - Identical to ray differentials
- Reconstruction
  - 1st order approximation

$$\mathbf{R}(\lambda + \Delta\lambda) - \mathbf{R}(\lambda) \approx \Delta\lambda \frac{\partial \mathbf{R}(\lambda)}{\partial \lambda}$$

APPLICATION

- Tracing
  - Identical to ray differentials
- Reconstruction
  - 1st order approximation

$$\mathbf{R}(\lambda + \Delta\lambda) - \mathbf{R}(\lambda) \approx \Delta\lambda \frac{\partial \mathbf{R}(\lambda)}{\partial \lambda}$$

$\mathbf{p} - \Delta\mathbf{p}$ $\mathbf{p}$ $\mathbf{p} + \Delta\mathbf{p}$

$\Delta\lambda$

Regular sampling

Stochastic sampling

RESULTS

Light Tracing (Arvo; 1986)

Stochastic - 2 minutes

Ours - 2 minutes

Stochastic - 60 minutes

RESULTS

Light Tracing (Arvo; 1986)

Stochastic - 2 minutes   Ours - 2 minutes   Stochastic - 60 minutes

RESULTS

Light Tracing (Arvo; 1986)

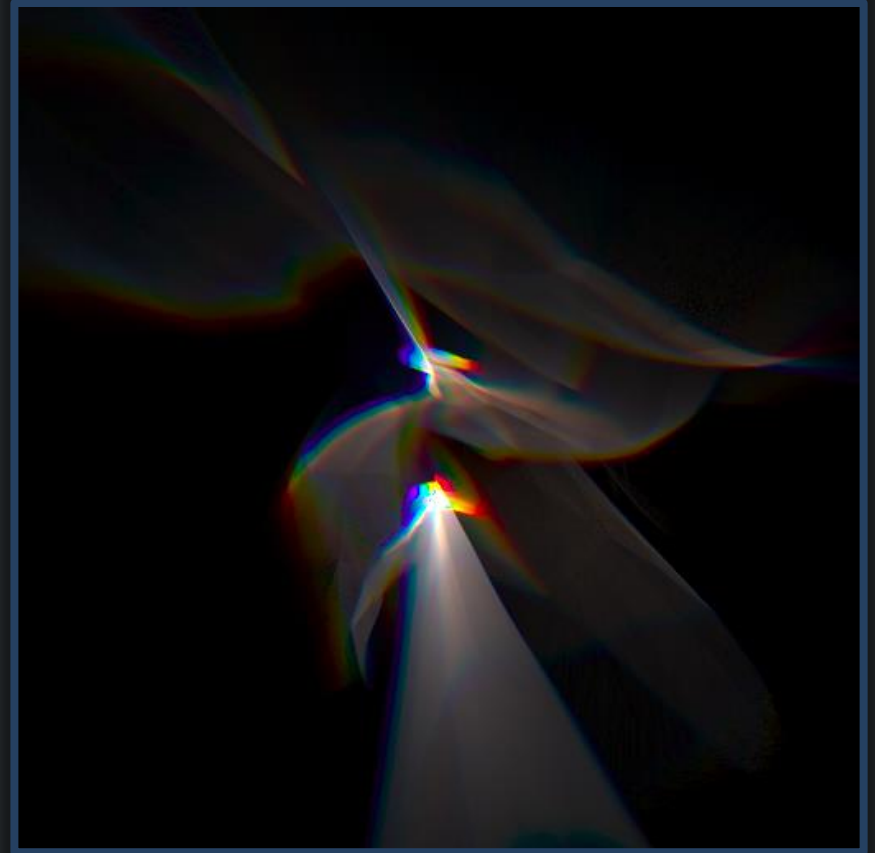Stochastic - 2 minutes   Ours - 2 minutes   Stochastic - 60 minutes

RESULTS

Light Tracing (Arvo; 1986)

Stochastic - 1 minute / frame

Ours - 1 minute / frame

RESULTS

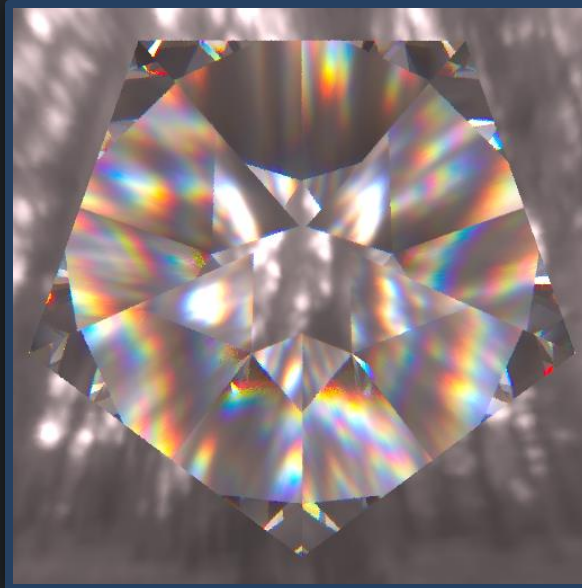## Light Tracing (Arvo; 1986)
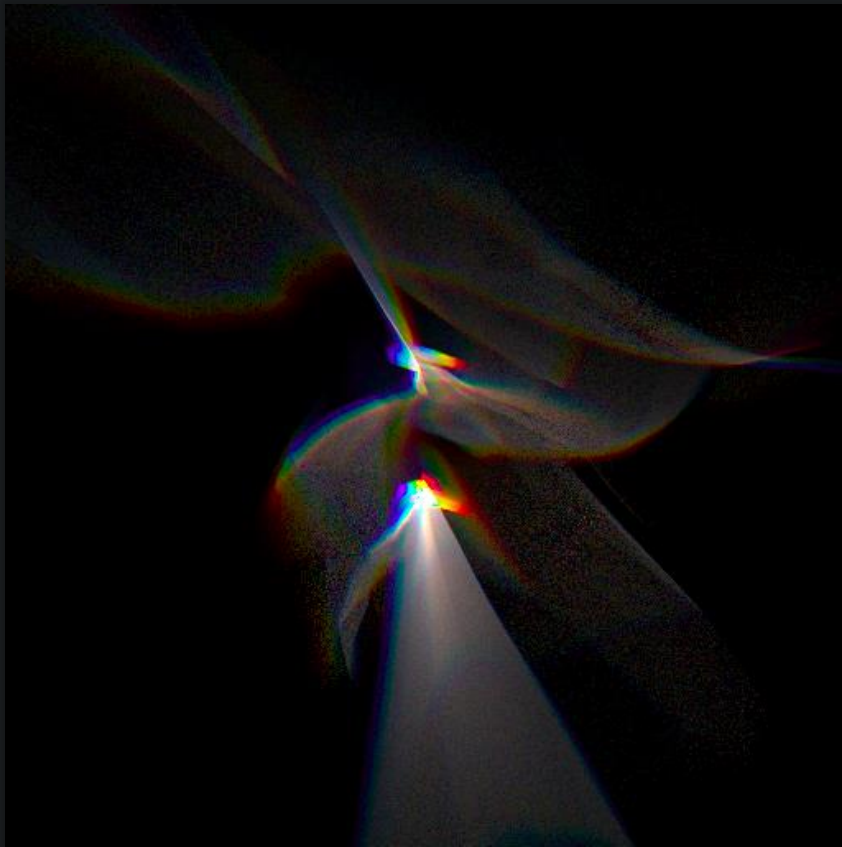
Stochastic - 1 minute / frame

Ours - 1 minute / frame

RESULTS

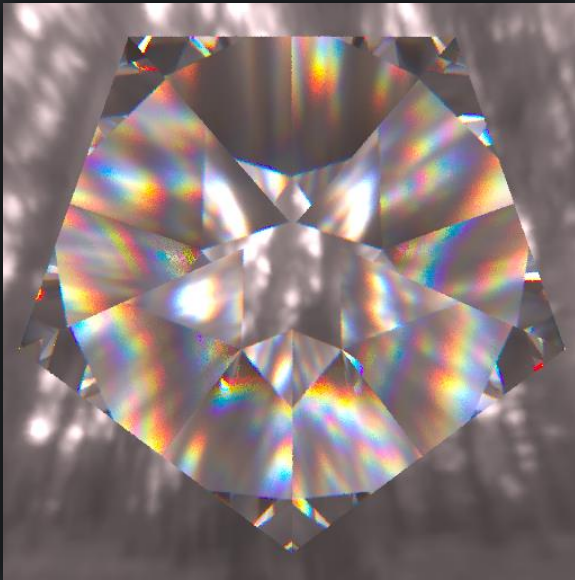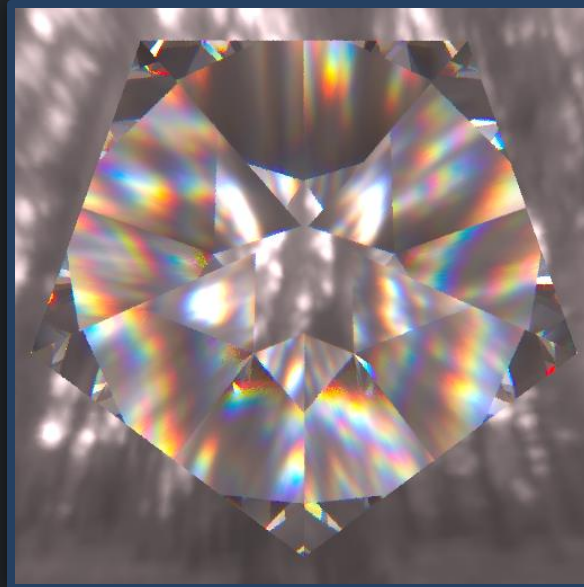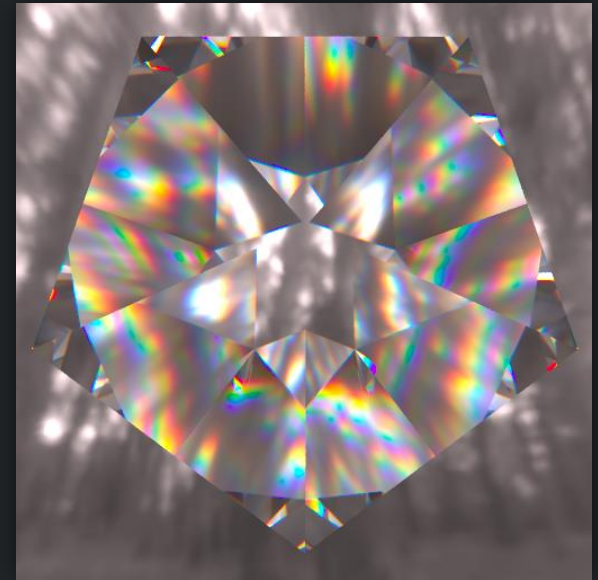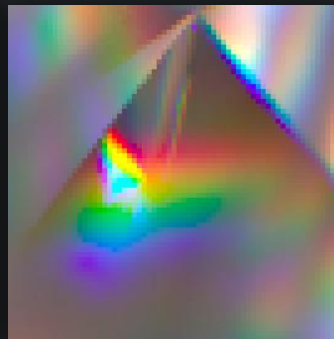## Eye Tracing (Thomas; 1986)
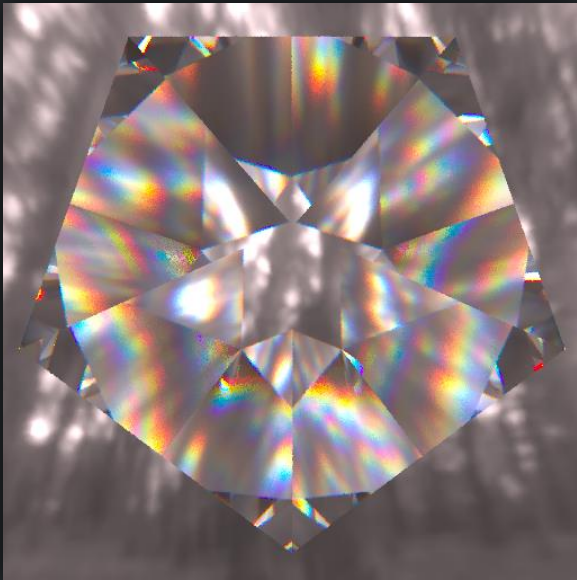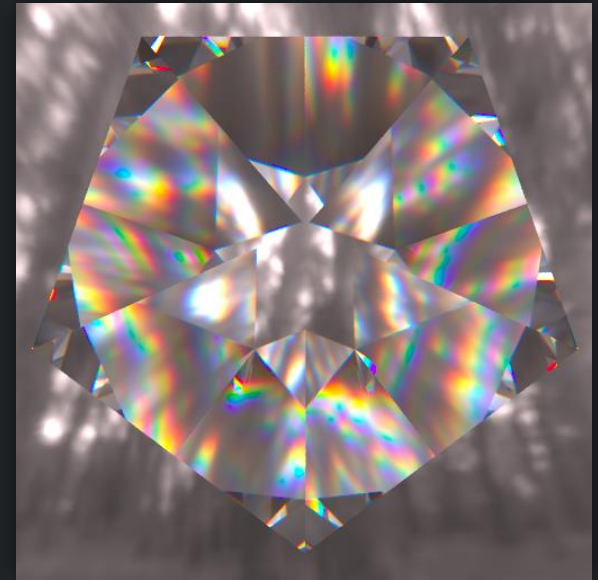
Stochastic – 1 second

Ours - 1 second

Stochastic - 20 seconds

RESULTS

Eye Tracing (Thomas; 1986)
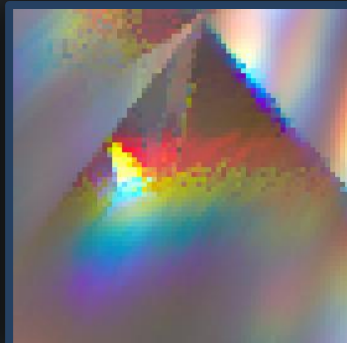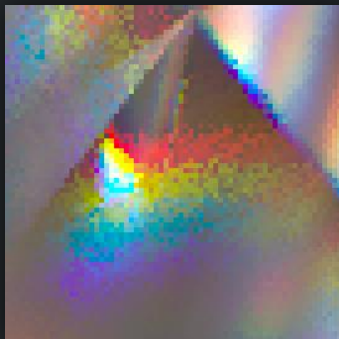

Stochastic – 1 second　　　　　Ours - 1 second　　　　　Stochastic - 20 seconds

RESULTS

Eye Tracing (Thomas; 1986)
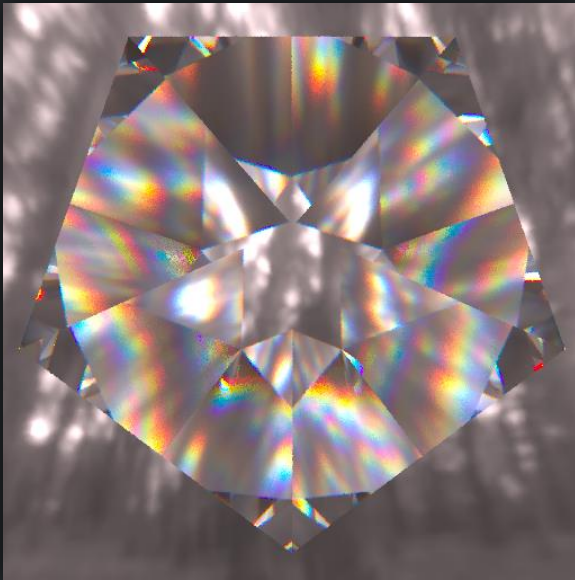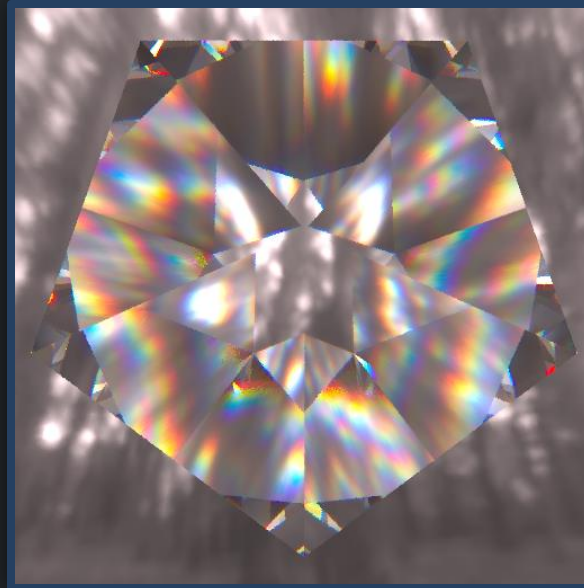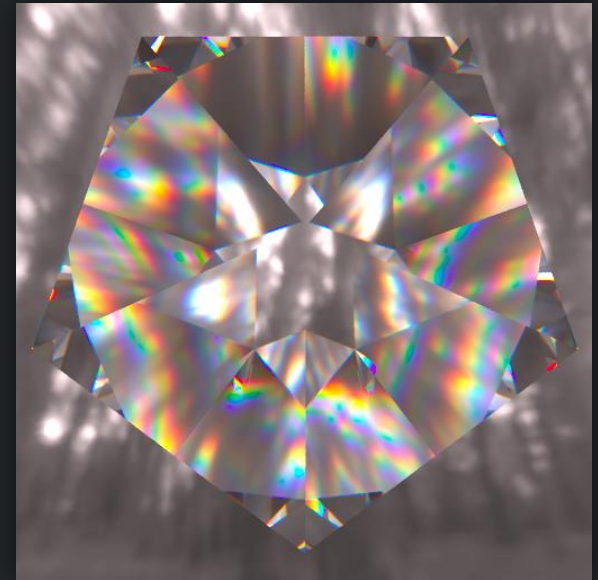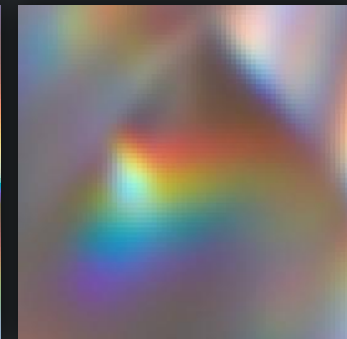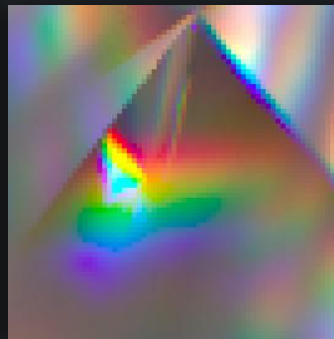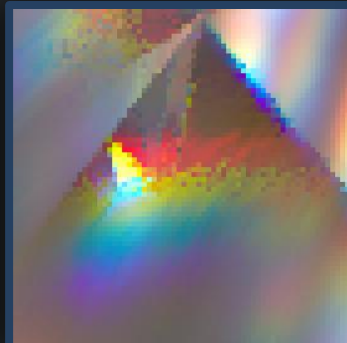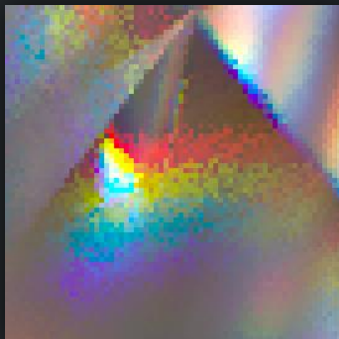

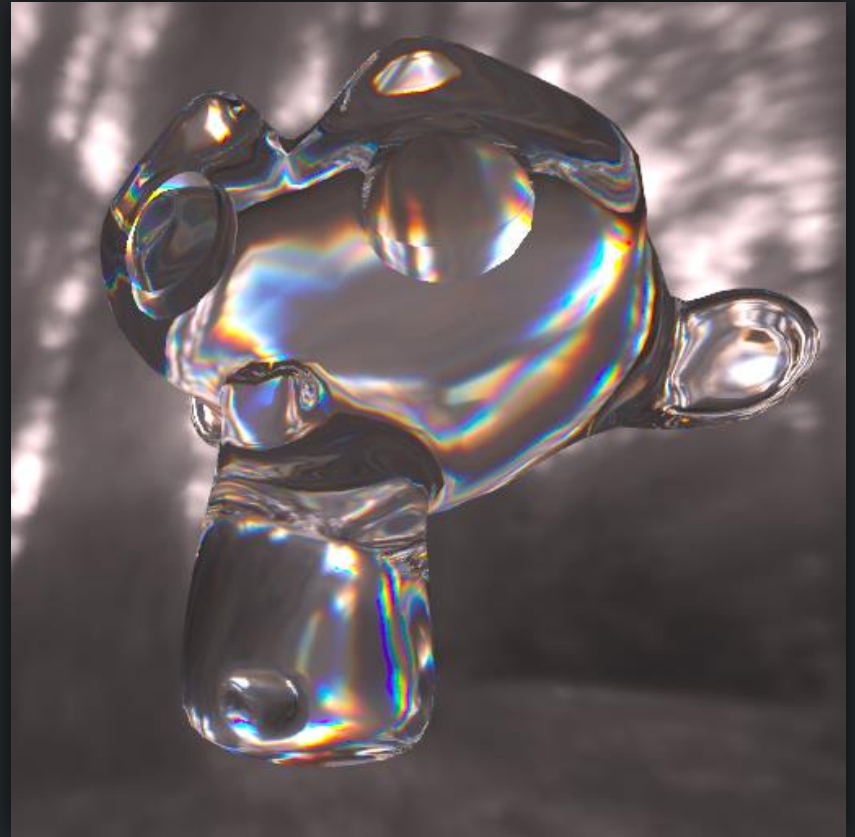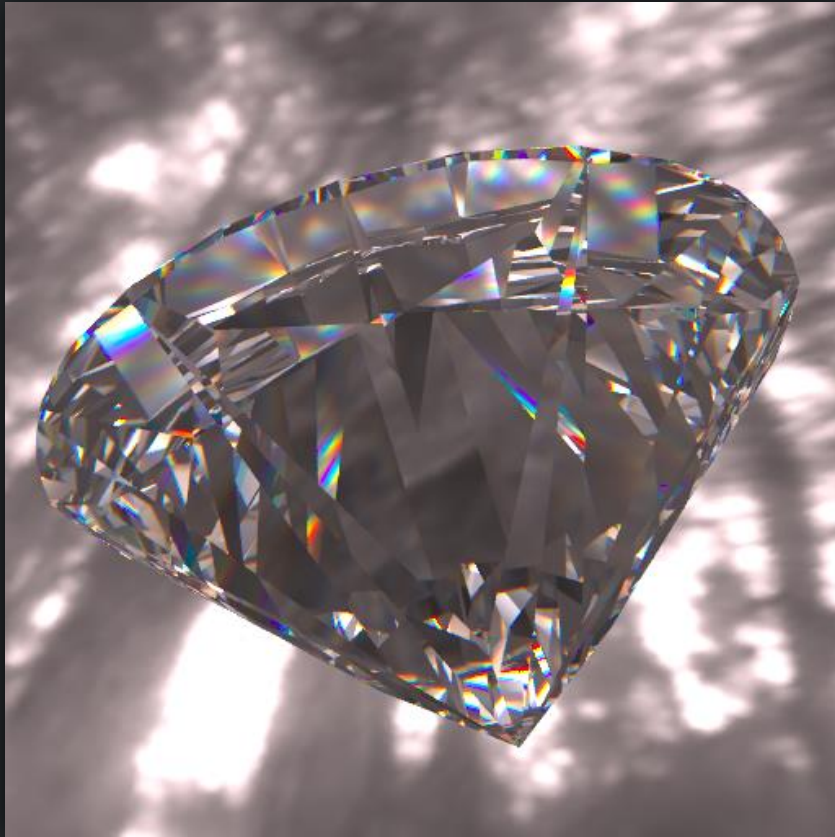Stochastic – 1 second · Ours - 1 second · Stochastic - 20 seconds
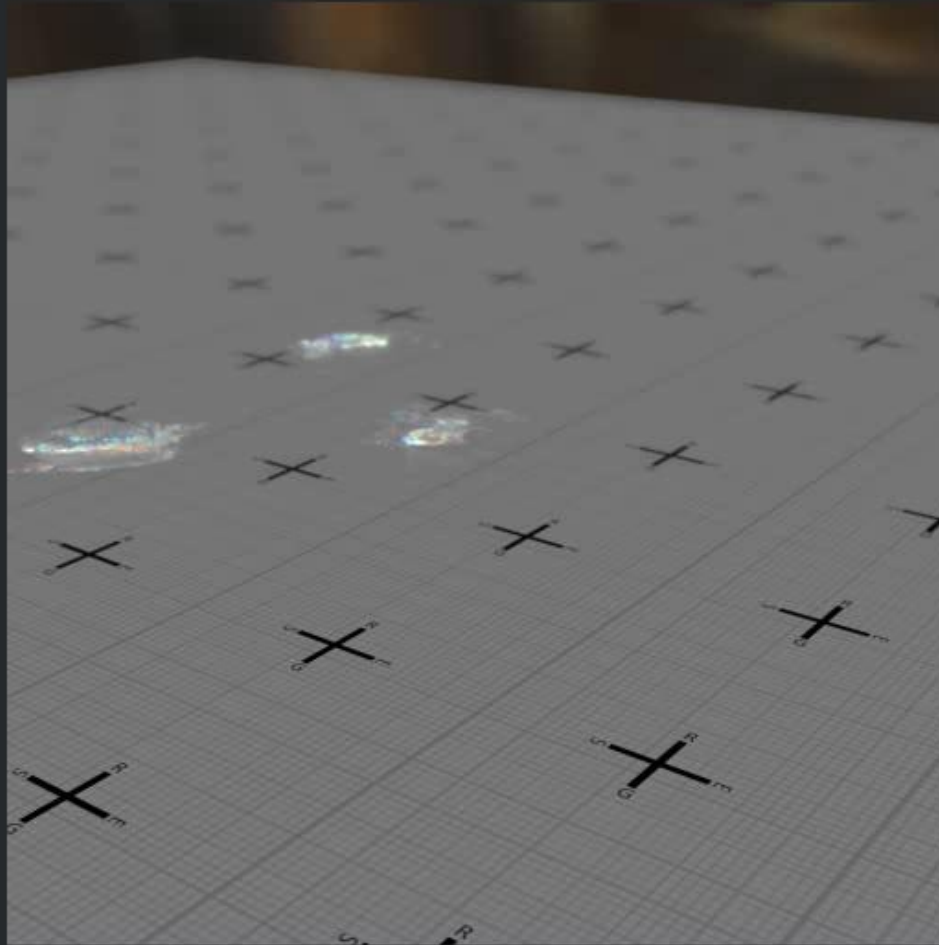
RESULTS

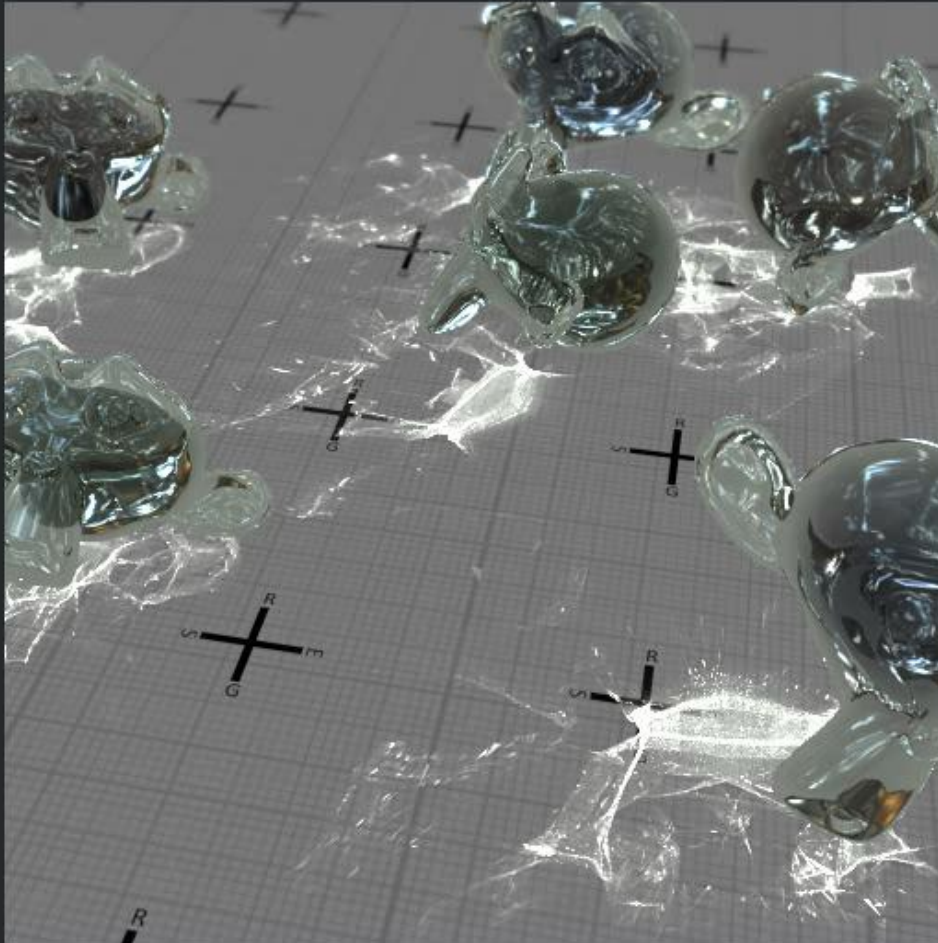Eye Tracing (Thomas; 1986)

RESULTS

Real-time photon mapping (Wyman, Davis; 2006)

Photon splatting – 25 Hz

RESULTS

Interactive on-screen caustic editing
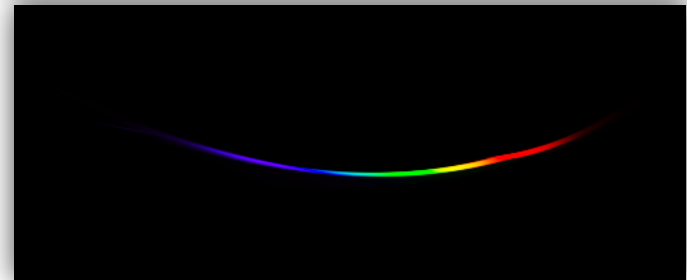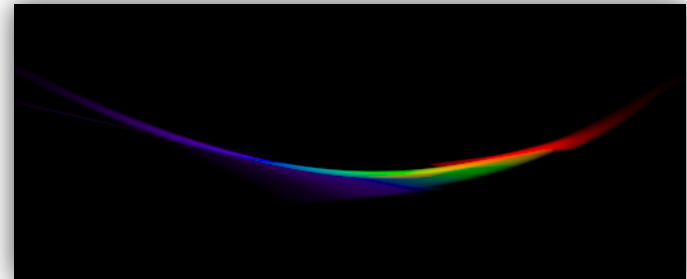

Interactive & local
dispersion control

CONCLUSION

- Spectral Ray Differentials

- Extend the happy family of ray differentials

- Reconstruction / variance reduction method

CONCLUSION

- Spectral Ray Differentials

- Extend the happy family of ray differentials

- Reconstruction / variance reduction method

- In future
  - Making SRD progressive

CONCLUSION

- Spectral Ray Differentials

- Extend the happy family of ray differentials

- Reconstruction / variance reduction method

- In future

  - Making SRD progressive

  - Application to other phenomena
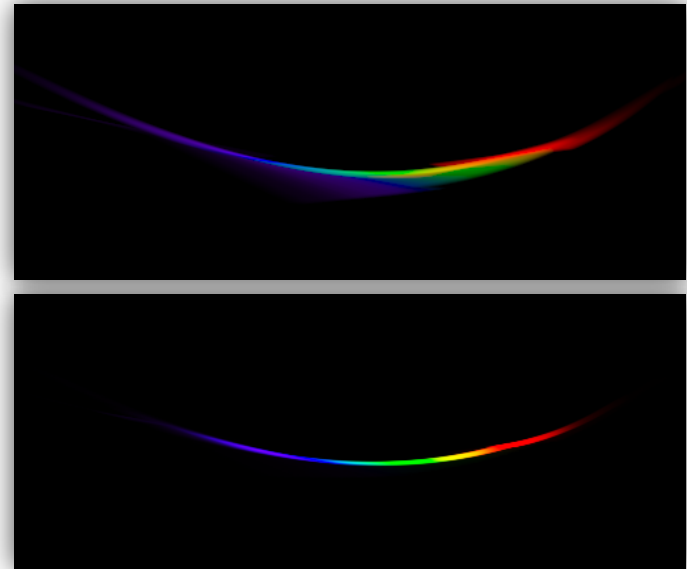
SPECTRAL RAY DIFFERENTIALS

Oskar Elek [1,2,3]   Tobias Ritschel [1,2,3]   Pablo Bauszat [4]

Marcus Magnor [4]   Hans-Peter Seidel [1,3]

Thanks

- Antti Oulasvirta, Oliver Klehm,
  Alexander Wilkie, Karol Myszkowski

- Anonymous reviewers

- You!

More info

- tinyurl.com/**SpectralRayDifferentials**